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The Uniqueness of God in Anselm's *Monologion*

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Abstract

In this paper, Anselm's argument for the uniqueness of God or, more precisely, something through which everything that exists has its being (*Monologion* 3) is reconstructed. A first reading of the argument leads to a preliminary reconstruens with one major weakness, namely the incompleteness of a central case distinction. In the successful attempt to construct a more tenable reconstruens some additional premises which are deeply rooted in an Anselmian metaphysics are identified. Anselm's argument seems to depend on premises such as that if two things have the same nature, then there is one common thing from which they have this nature and in virtue of which they exist. Furthermore it appears that infinite regresses are excluded by the premise that if everything that exists is through something, then there is something through which it is "most truly".

1. Introduction

Many modern philosophers and theologians from around the world are interested in St. Anselm's doctrine of God, although we live more than 900 years after this doctrine was constructed. Of key interest are the existence proofs: that God, or a being that fulfills a certain description B of a divine being, exists. In addition to this question of existence, there is also the question of uniqueness, that is, the question of whether there is no more than one being that falls under B (i. e., whether Bx and By imply x=y). It seems to me that the uniqueness question is strongly underexposed in the current literature.[1]

There are reasons for this underexposure. One reason may be that the existence of God is much more important for humanity's quest for the first or ultimate in general or for the foundations of rational theology in particular. Furthermore, it seems that existence is, in some sense, prior to uniqueness: if something does not exist, the question of whether, at most, one such thing exists seems completely superfluous. Finally, uniqueness may be trivial when a divine description B is used that consists of predicates that obviously imply uniqueness, such as the description "that which is greater than everything else". Because the greater-than relation is asymmetric, there can be no more than one such thing.[2]

In contrast, one might argue that divine uniqueness is theoretically quite important. For example, it may be conceived as a criterion of adequacy for monotheistic descriptions of

[1] Regarding uniqueness conditions for descriptions of God and their importance for (rational) theology, see Siegwart 2004 and Siegwart 2001.

[2] See, for example, Gaunilon's report of Anselm's *Proslogion* II proof, which Gaunilon presents in *Liber pro insipiente* I. There, he uses the phrase "that which is greater than all beings". But if there were two (that are greater than all beings), they would have to be greater than one another – and this contradicts the asymmetry of "greater than".

the divine: if a description B does not fulfill a uniqueness condition, it falls short of an adequate monotheistic description of God.

Furthermore, one can reasonably doubt that the controversy about God's existence will ever be theoretically settled (especially if one follows Kant in this respect). If this is indeed so, the uniqueness question has its own theoretical weight. There might be theoretically convincing arguments for uniqueness without there being theoretically convincing arguments for the existence of God.

The question of uniqueness can appear in several forms in theoretical discourse about God. For example, it can appear as part of the explicit question of what justifies the use, by authors such as Anselm of Canterbury, of definite descriptions such as "that than which nothing greater can be conceived". Successfully using definite descriptions usually presupposes that there is at most one object that fulfills the descriptive phrase. The question of uniqueness might also appear in the context of the question of whether the expression "God" can be used as an individual constant, if that means successful use, or whether the expression "God" has a unique referent, if using it is an attempt that can also fail. Finally, the uniqueness question can appear as an aspect of the question of whether the being whose existence is proven in an ontological argument is identical to the Christian God whom Anselm, in the *Proslogion* [=P], explicitly addresses in prayer. It seems a matter of course that there is one and only one God according to Christian self-understanding. Hence, it is necessary that a theoretical conception of God that is intended to capture the Christian conception of God has the same extension as the Christian conception: that there is at most one being that fulfills the theoretical conception.

There is no doubt that Anselm holds the uniqueness of God. It is less clear, however, what the systematic function of this theorem is in the construction of his doctrine of God. What are the logical relations between uniqueness and other theorems of Anselm's philosophical theology?

Although Anselm does not present an explicit argument for the uniqueness of God in P (where Anselm presents his famous so-called 'ontological argument'),[3] there are several such arguments at the beginning of the *Monologion* [=M]: an explicit argument for the uniqueness of that from which everything has its goodness (M1); an analogous but more implicit than explicit argument for the uniqueness of the greatest in the sense of the best (M2); and, finally, an elaborate argument for the uniqueness of that by virtue of which everything existing has its being (M3). This last argument is scrutinized in this paper by means of submitting it to the process of logical reconstruction. In this way, questions concerning the truth-preservingness of the inferences and the truth of the premises are much better tractable, provided that the formal reading is hermeneutically tenable. So, instead of using the text merely as a source of inspiration, we aim for a reconstruction that is both charitable and accurate with respect to Anselm's argument and its merits. As we will see, however, a preliminary reconstruction can call for a revision by both the demands of charity and the demands of accuracy.

[3] However, one may formulate a uniqueness proof "in the spirit of *Proslogion*"; see Tapp 2012.

2. The reconstruendum: text, structure, and aim of proof

2.1 *The text*

The beginning of M3 reads:[4]

(S1)	*⟨Denique non solum omnia bona per idem aliquid sunt bona, et omnia magna per idem aliquid sunt magna, sed⟩ quidquid est, per unum aliquid videtur esse.*	⟨Furthermore, not only is it the case that all good things are good through one and the same thing and all great things are great through one and the same thing, but also⟩ whatever exists, apparently exists through one thing.
(S2)	*Omne namque quod est, aut est per aliquid aut per nihil.*	For everything that exists, exists either through something or through nothing.
(S3)	*Sed nihil est per nihil.*	But nothing exists through nothing.
(S4)	*Non enim vel cogitari potest, ut sit aliquid non per aliquid.*	For it is impossible even to conceive of something existing through nothing.
(S5)	*Quidquid est igitur, non nisi per aliquid est.*	Whatever exists, then, exists only through something.
(S6)	*Quod cum ita sit, aut est unum aut sunt plura, per quae sunt cuncta quae sunt.*	Now, because this is the case, there is either one or more than one thing through which all existing things exist.
(S7)	*Sed si sunt plura, aut ipsa referuntur ad unum aliquid, per quod sunt, aut eadem plura singula sunt per se, aut ipsa per se invicem sunt.*	But if there are more than one, then they are either themselves reducible to some one thing through which they exist, or each of them exists individually through itself, or they exist mutually through one another.
(S8)	*At si plura ipsa sunt per unum, iam non sunt omnia per plura, sed potius per illud unum, per quod haec plura sunt.*	But, if these things exist through one thing, then all things do not exist through more than one, but rather through that one thing through which the more than one exist.
(S9)	*Si vero ipsa plura singula sunt per se, utique est una aliqua vis vel natura existendi per se, quam habent, ut per se sint.*	If, however, the more than one exist individually, each through itself, there is, at any rate, some power or property of existing through oneself (*existendi per se*), that each possesses such that it is through itself.
(S10)	*Non est autem dubium, quod per id ipsum unum sint, per quod habent, ut sint per se.*	But then, doubtless, they would exist through this one thing through which they possess the capacity to exist through oneself.
(S11)	*Verius ergo per ipsum unum cuncta sunt, quam per plura, quae sine eo uno esse non possunt.*	More truly, then, do all things exist through this one thing than through those more than one, which, without it, cannot exist.
(S12)	*Ut vero plura per se invicem sint, nulla patitur ratio, quoniam irrationabilis cogitatio est, ut aliqua res sit per illud, cui dat esse.*	But that these things exist mutually through one another, no reason can admit; because it is an irrational conception that something should exist through a thing to which it gives existence.
(S13)	*Nam nec ipsa relativa sic sunt per invicem.*	For not even relative things exist thus mutually through one another.
(S14)	*Cum enim dominus et servus referantur ad invicem, et ipsi homines, qui referuntur, omnino non sunt per invicem, et ipsae relationes quibus referuntur, non omnino sunt per invicem, quia eaedem sunt per subiecta.*	For, though the terms master and servant are used with mutual reference, the men thus spoken of do not exist through each other at all, nor do the relations themselves by which they are spoken of exist through each other at all, because these relations exist through the subjects.
(S15)	*Cum itaque veritas omnimodo excludat plura esse per quae cuncta sint, necesse est unum illud esse, per quod sunt cuncta quae sunt.*	Therefore, because truth altogether excludes the supposition that there is more than one thing through which everything exists, that being, through which all existing things exist, must be one.

[4] I have enumerated the Latin sentences (S1) to (S15). The angle brackets indicate that the first part of the first sentence is not part of the argument but is a bridge from the earlier arguments to this one. The Latin text is taken from the Franciscus Salesius Schmitt edition of M (Anselm 1964). The English translation is my own, based upon three major sources: Deane 1910, 41–42; Davies & Evans 1998, 13–14; Hopkins & Richardson 2000, 9–10. All three sources, however, have their advantages and disadvantages. Deane, for example, translates the first sentence, which proposes the aim of the proof, as "whatever is, apparently exists through something that is one and the same" (*quidquid est, per unum aliquid videtur esse*), emphasizing uniqueness more strongly than the original Latin text does. Davies & Evans add an enumeration ("first", "second", "third" in (S8), (S9), and (S12)) that has no counterpart in the original Latin text, and suppress the important "*potius*" in (S8). Hopkins & Richardson switch back and forth between "is" and "exists", and they suppress the Latin "*singula*" in (S7).

2.2 *The aim of the proof, disambiguation*

The argument starts in (S1), after a short bridge from the preceding arguments, by an exposition of the aim of the proof: "whatever exists, apparently exists through one thing" (*quidquid est, per unum aliquid videtur esse*). Similarly, the argument concludes in (S15), "that being, through which all existing things exist, must be one" (*unum illud esse, per quod sunt cuncta quae sunt*). If we assume that the same aim of the proof is stated in the exposition and conclusion, then we face a certain ambiguity here. What exactly does it mean that whatever is, is through one thing?

We may distinguish three readings of the aim of the proof:

a. (existence reading) There is an x such that whatever is has its being through x. This is a singular existence statement of the form "There is an x, such that for all y, …". It does not exclude the possibility that in addition to one example of such an x, there might be others.
b. (uniqueness reading) There is at most one x such that whatever is has its being through x (in contrast to being through a y, which is different from x). This is a universal statement of the form "If x would … and also y would …, then x=y".
c. (combined reading) There is one and only one x such that whatever is has its being through x. This is the conjunction of existence and uniqueness.

The conclusion in (S15) suggests the uniqueness reading by wording and by argumentative context. The wording is "that being … must be one" (*necesse est unum illud esse …*), and the last step to this conclusion is the explicit rejection of "the supposition that there is more than one thing …". Although the existence of such a being seems to go unconsidered in (S15), it seems to be an explicit part of the intended conclusion as exposed in (S1): "whatever exists, apparently exists through one thing" (*quidquid est, per unum aliquid* *videtur esse*). Hence, (S1) speaks in favor of a combined reading of the intended conclusion as including existence and uniqueness.

Closer inspection of the argument leads to strong support for the combined reading: there is one and only one x such that whatever is is through this x. This support comes from the logical structure of the overall argument and from the reading of the intermediate conclusion.

2.3 *The logical structure of the argument I*

The argument has two parts. The first part (S2–S5) reaches the intermediate conclusion that whatever exists does so through something (*quidquid est igitur, non nisi per aliquid est*, S5; the word *igitur* signals a conclusion), whereas the second part (S6–S15) is intended to show that there is only one being through which all existing things exist (*necesse est unum illud esse, per quod sunt cuncta quae sunt*, S15). In a sense, then, the first part comes close to an existence proof, whereas the second part is close to a uniqueness proof. This characterization, however, is only partially adequate because although the intermediate conclusion is a (conditionalized) existence claim, it is not the existence claim that is part of the overall aim of the proof. The intermediate conclusion is that whatever exists does so through something, whereas the existence component of the overall conclusion is that there is something through which every existing thing exists.

Here, two relations are used, the one-place relation of being and the two-place relation of being-through. I assign to them the one-place predicate E (for the Latin "… *est*") and the two-place predicate EP (for the Latin "… *est per*… "). The intermediate conclusion then reads,[5]

(S5) $\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$.

The second part of the argument (S6–S15), with the aim of showing that there is at most one thing through which all that is is, consists mainly in an indirect proof or a reductio argument for one member of a disjunction. According to (S6), either there is one being or there are more than one through which all existing things exist (*aut est unum aut sunt plura* …). In (S15), the second disjunct is regarded as refuted (*veritas omnimodo excludat plura esse* …). Hence, the second disjunct – that it might have been more than one – is taken to have been reduced to the absurd between (S6) and (S15). The reductio is organized in (S7) by a threefold case distinction. If there are several things (*plura*) through which all existing things exist, one of the following cases is said to hold:

a) the several things can be reduced to a single one;
b) each one is through itself; or
c) they are mutually through one another.

The rest of the second part of the argument consists of reductiones for each of these cases: first, case a) in (S8); then, case b) in (S9–S11); and, finally, case c) in (S12–S14).

[5] In their contributions to this volume, Friedrich Reinmuth and Geo Siegwart point out that in a logical reconstruction, strictly speaking, expressions of formal language are assigned to expressions of natural language. However, once this relationship is clarified, it might be allowed to stay within the loose manner of speaking according to which the formal expressions "mean" something expressed in natural language or that the intermediate conclusion has a formal logical "reading". See Reinmuth 2014, Siegwart 2014.

2.4 *Disambiguation of the main disjunction*

The main disjunction in (S6) contains a further ambiguity: how should we understand the alternative to uniqueness, namely, the case that there is more than one through which all existing things exist? Is the alternative that there are several beings that universally provide existence, so there are several y such that for each y it is true that all that is is through y? Or is the alternative that there are several y that distributively provide existence, that is, there are several y such that for each one there is at least one existing x such that x is through it?

This alternative can be noted more handily by writing it in the language of set theory (although we will not use set theoretic means in the reconstruction). Is the main disjunction a question of the number of elements of the set P_U ["U" for "universally"] with

$$P_U := \{y \mid \forall x\ (Ex \rightarrow EP(x,y))\}$$

or of the set P_D ["D" for "distributively"] with

$$P_D := \{y \mid \exists x\ (Ex \wedge EP(x,y))\}?$$

Without using set theoretic language, the question is to decide between the two following readings of the main disjunction in (S6), "*aut est unum aut sunt plura, per quae sunt cuncta quae sunt*":

($S6_U$) $\forall y_1,y_2\ (\forall x\ (Ex \rightarrow EP(x,y_1)) \wedge \forall x\ (Ex \rightarrow EP(x,y_2)) \rightarrow y_1{=}y_2) \vee$
$\exists y_1,y_2\ (\forall x\ (Ex \rightarrow EP(x,y_1)) \wedge \forall x\ (Ex \rightarrow EP(x,y_2)) \wedge y_1{\neq}y_2)$

or

($S6_D$) $\forall y_1,y_2\ (\exists x\ (Ex \wedge EP(x,y_1)) \wedge \exists x\ (Ex \wedge EP(x,y_2)) \rightarrow y_1{=}y_2) \vee$
$\exists y_1,y_2\ (\exists x\ (Ex \wedge EP(x,y_1)) \wedge \exists x\ (Ex \wedge EP(x,y_2)) \wedge y_1{\neq}y_2)$.

According to an intuitive first grasp, the universal reading seems slightly more convincing. In the end, the argument is about God. It is not about a demiurge with a strongly restricted field of responsibility but about the universal source of being. Furthermore, (S6) follows immediately after the conclusion of the first part of the argument (S5), according to which all that is is through something:

(S5) $\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$.

This formula seems to fit better with the argumentative context because it is part of the reading ($S6_U$).

Closer inspection of the argumentative context, however, reveals strong evidence against the universal and in favor of the distributive reading. First, according to the universal reading, we face a gap in the proof step from (S5) to (S6), which has the form of an unjustified change of quantifiers. Having reached

(S5) $\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$

or the equivalent

(*) $\forall x\ \exists y\ (Ex \rightarrow EP(x,y))$,

($S6_U$) would address the question of the uniqueness of the y's in

(**) $\exists y\ \forall x\ (Ex \rightarrow EP(x,y))$.

Whereas (S5) reaches a thesis with the logical form $\forall x \exists y \ldots$, ($S6_U$) would discern two types of existence examples for a thesis with the logical form $\exists y \forall x \ldots$, and such a transfer of quantifiers is, in general, inadmissible.

Further evidence comes from considering the step from (S6) to (S7). The second disjunct of ($S6_U$) would come down to assuming two different, universally existence-providing objects y_1 and y_2 (i. e., objects fulfilling $\forall x\ (Ex \rightarrow EP(x,\_\_))$). The instantiation of the property of being universally existence-providing to y_1 and y_2 would give $EP(y_1,y_1)$, $EP(y_2,y_2)$, $EP(y_2,y_1)$, and $EP(y_1,y_2)$, which means several of the properties according to which the case distinction in (S7) proceeds. Hence, one of the cases would have sufficed, and the case distinction would have turned out to be superfluous. Therefore, the fact that there is this argument by case distinction, in which the cases are clearly distinguished by "or" (*aut*), is further evidence against the universal reading of (S6).

Therefore, we take the distributive reading of (S6), ($S6_D$), to be the correct one. We can introduce a handy shorthand version of it by observing that Anselm refers several times to the (possibly) "more than one" or "several" (*plura*) through which all that is is. Let *P* be the predicate with the extension of the *plura per quae cuncta sunt* in the distributive reading:

$$Py :\leftrightarrow \exists x\ (Ex \wedge EP(x,y)).$$[6]

With this shorthand notation, (S6) finally reads as follows:

($S6_D$) $\forall y_1,y_2\ (Py_1 \wedge Py_2 \rightarrow y_1{=}y_2) \vee$
$\exists y_1,y_2\ (Py_1 \wedge Py_2 \wedge y_1{\neq}y_2)$.

2.5 *The logical structure of the argument II*

The logical structure of the whole argument proceeds from the main disjunction in (S6) by reducing the second disjunct to the absurd. All three cases of the case distinction in (S7) are reduced to the absurd. Hence, at the end of the second part of the argument, the first disjunct of ($S6_D$) is established, namely

(S15) $\forall y_1,y_2\ (Py_1 \wedge Py_2 \rightarrow y_1{=}y_2)$.

From (S5) and (S15), we can derive the targeted existence-and-uniqueness claim as follows. Take, as a starting point, the claim that something whatsoever exists (i. e., that there is an x_1 with Ex_1). Anselm would surely suppose so. According to (S5), we find a distributively existence-providing y_1 such that $EP(x_1,y_1)$. This y_1 must then already be universally existence-providing because the assumption that there is another y_2 that is particularly existence providing (i. e., that there is an x_2 such that Ex_2 and $EP(x_2,y_2)$), by (S15) implies $y_1{=}y_2$. The same argument also shows, by means of (S15), that y_1 is unique in universally providing existence.

Therefore, the overall aim of the proof reads

(S1/15) $\exists! y\ \forall x\ (Ex \rightarrow EP(x,y))$,[7]

[6] If it is certain that EP(x,y) implies Ex, one might, of course, simplify this definition to $Py :\leftrightarrow \exists x\ EP(x,y)$.

[7] The $\exists! y$ quantifier in front of a formula $\phi(y)$ means there is exactly one y such that $\phi(y)$, that is, $\exists! y\ \phi(y)$ is shorthand for $\exists y\ (\phi(y) \wedge \forall z\ (\phi(z) \rightarrow z{=}y))$.

and this aim is realized in two steps: in the first part of the argument for the intermediate conclusion

(S5) $\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$

and in the second part of the argument with the conclusion

(S15) $\forall y_1,y_2\ (Py_1 \wedge Py_2 \rightarrow y_1{=}y_2)$.

The fact that (S15) is the end of both the whole argument and the second part of it also explains why Anselm's formulation of (S15) sounds like a uniqueness claim, although the entire argument targets an existence-and-uniqueness claim.

We now must analyze the two parts of the argument in more detail.

2.6 *Omitting the first part of the argument (S2–S5)*

Even a superficial analysis makes it clear that the first part of the argument is not quite substantial: All that is must be through something or through nothing, but it is inconceivable to be through nothing, so … In this argument, a particular form of *tertium non datur* for existing things is presupposed: they must either be through something or through nothing. Anselm does not explain what it means to be through nothing, but he will certainly exclude it because it contradicts the proposition (S3) that nothing exists through nothing (*sed nihil est per nihil*).

It seems that Anselm wants to justify proposition (S3) by (S4): "For it is altogether inconceivable that anything should not exist by virtue of something" (*non enim vel cogitari potest, ut sit aliquid non per aliquid*; justification indicated by signal word *enim*). However, it is doubtful whether founding (S3) upon a stronger, modalized version of itself (it is not the case that … because it is inconceivable that …), which is not justified any further, is a reasonable justification at all. Hence, it may be more plausible to conceive of (S4) as an explanation of (S3) rather than a justification proper.

Most likely, by "*nihil est per nihil*" Anselm refers to the theorem from the antique philosophy of nature that "*de nihilo nihil fit*". Aquinas, however, notes later that this theorem is valid only insofar as the research interests of philosophers of nature were in (generalizations of) single changes in nature.[8]

We leave the first part of the argument unreconstructed and treat its conclusion (S5), that all that is is through something, in our reconstruction simply as a premise. As such, it then displays the first part of (S6), in which Anselm explicitly recurs to (S5) when he says, "Because this [= what was claimed in S5], is true …" (*quod cum ita sit* …).

[8] To be more precise, Aquinas' theorem holds for changes in which something remains what it is (for instance, if a sheet of metal is hammered) and for changes in which something becomes something else (for instance, burning something to ashes), but not for "changes" such as when something comes into existence out of nothing. See *Summa Contra Gentiles* II,16,14.

2.7 *Preliminary result*

So far, our analysis has proposed the following structure of the overall argument:

(S1)–(S5)	*[…] Quidquid est igitur, non nisi per aliquid est.*	First part of the argument. Intermediate conclusion: all that is is through something.
(S6)	*Quod cum ita sit, aut est unum aut sunt plura, per quae sunt cuncta quae sunt.*	Main disjunction: either it is one (left disjunct) or more than one (right disjunct) through which all that is is.
(S7)	*Sed si sunt plura,* *aut ipsa referuntur ad unum aliquid, per quod sunt,* *aut eadem plura singula sunt per se,* *aut ipsa per se invicem sunt.*	Case distinction in case of the right disjunct: a) the several things can be reduced to a single one or b) each one is through itself or c) they are each through itself.
(S8)	*At si plura ipsa sunt per unum, …*	Case a) is reduced to the absurd
(S9)–(S11)	*Si vero ipsa plura singula sunt per se, …*	Case b) is reduced to the absurd
(S12)–(S14)	*Ut vero plura per se invicem sint, …*	Case c) is reduced to the absurd
(S15)	*Cum itaque veritas omnimodo excludat plura esse per quae cuncta sint,* *necesse est unum illud esse, per quod sunt cuncta quae sunt*	Result of the case distinction: the right disjunct is absurd in every case. Application to the main disjunction: the left disjunct must be true ⇒ conclusion.

With respect to the conclusiveness of this argument, two questions are important:

1. *whether and how* the single cases are (successfully) reduced to the absurd and
2. *whether* the case distinction is complete.

Generally, both questions depend on each other like a system of communicating tubes: the less specific a case condition is, the more instances it covers; hence, the better for the completeness of the case distinction, but the worse for deriving a contradiction from the less specific case conditions. In contrast, if the case conditions are very narrow, the contradiction might be easier to derive, but the completeness of the case distinction becomes less sure.

3. Kernel of the argument I: reducing the three cases to the absurd

Having reached the intermediate conclusion, Anselm explicitly asks for the uniqueness of that which provides existence, hence, the question of how many beings fall under the predicate P: either it is one, or there are more than one through which all that is is (*aut est unum aut sunt plura, …*).[9] The left member of this disjunction is the aim of the second part of the proof. Hence, the right member is to be excluded. To that end, a case distinction is introduced in (S7):

[9] That P is no empty concept, i. e., that there is something that gives being at all, is not discussed by Anselm. However, it follows immediately from the trivial observation that something exists ($\exists x\ Ex$) by applying the intermediate conclusion that all existing things exist through something ($\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$). Therefore, $\exists y\ \exists x\ EP(x,y)$, that is, P is "inhabited".

(S7) However, if there are more than one, either these are themselves to be referred to some one being, through which they exist, or they exist separately, each through itself, or they exist mutually through one another.

"If there are more than one" – that is, according to our reading of (S6), we have y_1 and y_2 such that $Py_1 \wedge Py_2 \wedge y_1 \neq y_2$ – then one of the following three cases holds:

(S7a) "these are themselves to be referred to some one being, through which they exist": $\exists z\ (EP(y_1,z) \wedge EP(y_2,z))$ or
(S7b) "they exist separately, each through itself": $EP(y_1,y_1) \wedge EP(y_2,y_2)$ or
(S7c) "they exist mutually through one another": $EP(y_1,y_2) \wedge EP(y_2,y_1)$.

By combining parts (S7a–c), we obtain the following reconstruction of (S7):

(S7) $\forall y_1,y_2\ (Py_1 \wedge Py_2 \wedge y_1 \neq y_2 \rightarrow$
$\exists z(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1)))$.

Because Anselm addresses the three cases differently, we will consider them separately in the following subsections.

3.1 *Case a): a common source of being*

In case a), Anselm argues as follows:

(S8) "But, if these beings exist through one being, then all things do not exist through more than one, but rather through that one being through which these exist."

This step can, once again, best be reconstructed point-by-point:

(S8a) So, "if these beings exist through one being", that is, there is a z such that our y_1 and y_2 with Py_1, Py_2 and $y_1 \neq y_2$ are through z: $\exists z\ (EP(y_1,z) \wedge EP(y_2,z))$,
(S8b) "then all things do not exist through more than one", that is, all x_1 and x_2, which are through y_1 und y_2, are not through y_1 and y_2: $\neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$
(S8c) "but [all things are] rather through that one being through which these exist", that is, x_1 and x_2 are through the z through which y_1 and y_2 are: $EP(x_1,z) \wedge EP(x_2,z)$.

In moving from (S8b) to (S8c), Anselm obviously makes use of the transitivity of EP: the x's[10] stand in relation EP to the y's and the y's to z. This should imply that the x's stand in EP to z:

$$EP(x_i,y_i) \wedge EP(y_i,z) \rightarrow EP(x,z).$$

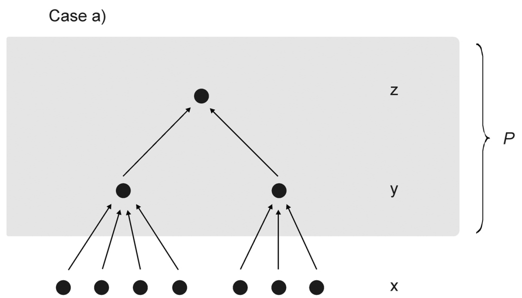
[10] For the sake of simplicity, I will adhere to the customary abuse of language and talk, for example, as if variables would be sortal terms.

This is an instantiation of the transitivity of EP:

$$\forall x,y,z\ (EP(x,y) \wedge EP(y,z) \rightarrow EP(x,z)).$$

Hence, x_1 and x_2 are *not so much* through y_1 and y_2 but *"rather"* through one, z. (We will discuss later how this "not so much" / "rather" can be understood.)

We have found a further element z through which y_1 and y_2, and therefore, by transitivity, x_1 and x_2 are. Hence, we are in a situation like that in the following illustration where the EP relation between x and y is symbolized by an arrow from x to y:



In this situation, Anselm concludes that all "*iam non sunt per plura, sed potius per illud unum, per quod haec plura sunt*". The x's are not through the *plura*, y, but rather (*potius*) by this one z, through which the y's are.

From the perspective of formal logic, this is not easily reconstructible. What does it mean that "at first" the relation EP holds between x and y, and "then" it does not do so anymore, or at least that "then" it holds "rather" between the x's and z? Classical truth does not allow "more" or "less" of truth besides "true" and "false". Hence, we must conceive of "A rather than B" as "$A \wedge \neg B$".

This means that from EP(x,y) and EP(y,z) we first obtain EP(x,z) by transitivity and then conclude ¬EP(x,y) from it. The question is what the exact form of the implicitly used inference principle is that covers this move from EP(x,y), EP(y,z) and EP(x,z) to ¬EP(x,y). It cannot reasonably be

$$\forall x,y,z\ (EP(y,z) \wedge EP(x,z) \rightarrow \neg EP(x,y))$$

because for y=z, this implied $EP(y,y) \wedge EP(x,y) \rightarrow \neg EP(x,y)$, and hence $EP(y,y) \rightarrow \neg EP(x,y)$. That cannot be Anselm's intention because then nothing could be through something which is through itself – that is, nothing could be through God! Therefore, at least, we have to exclude y=z, and we therefore arrive at the following principle,

(Excl) $\forall x,y,z\ (EP(y,z) \wedge EP(x,z) \wedge y \neq z \rightarrow \neg EP(x,y)).$

This principle claims something like the "exclusivity" of higher existence providers: if (the being of) y depends on a higher z, and if this dependence is "real", in that y is different from z, then nothing lower, x, can (really) depend on y because such a lower x would then, by transitivity, depend "rather" on z itself and hence "less" (i. e., not) on y. (Excl), however, is a problematic principle in the context of an Augustinian-inspired metaphysics such as Anselm's because the hierarchy of being is drastically reduced to only two levels: what provides existence and what does not provide existence (i. e., creator and creatures). We will return to this problematic reason, but here we will provisionally accept it to have a chance of obtaining a vertically intact part of the derivation at all.

The derivation of a contradiction in this case proceeds as follows:

0		$\forall y\ (Py :\leftrightarrow \exists x\ EP(x,y))$	(def(P))
1		$\exists y_1,y_2\ (Py_1 \wedge Py_2 \wedge y_1 \neq y_2)$	(right disjunct of S6)
2	(S8a)	$\exists z\ (EP(y_1,z) \wedge EP(y_2,z))$	(case assumption a)
3		$EP(x_1,y_1) \wedge EP(x_2,y_2)$	(from 0,1)
4		$EP(y_1,z) \wedge EP(y_2,z)$	(from 2)
5		$\forall x,y,z\ (EP(x,y) \wedge EP(y,z) \rightarrow EP(x,z))$	(Trans(EP))
6	(S8c)	$EP(x_1,z) \wedge EP(x_2,z)$	(from 3,4,5)
7		$\forall x,y,z\ (EP(y,z) \wedge EP(x,z) \wedge y \neq z \rightarrow \neg EP(x,y))$	(Excl)
8		$EP(y_1,z) \wedge EP(x_1,z)$	(from 4,6)
9		$y_1 \neq z \rightarrow \neg EP(x_1,y_1)$	(from 7,8)
10		$EP(y_2,z) \wedge EP(x_2,z)$	(from 4,6)
11		$y_2 \neq z \rightarrow \neg EP(x_2,y_2)$	(from 7,10)
12		$y_1 \neq z \vee y_2 \neq z \rightarrow \neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 9,11)
13		$y_1 \neq y_2 \rightarrow y_1 \neq z \vee y_2 \neq z$	(inequality axioms)
14		$y_1 \neq y_2 \rightarrow \neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 12,13)
15	(S8b)	$\neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 1,14)
16		$\perp$	(from 3,15)
17		$\exists z\ (EP(y_1,z) \wedge EP(y_2,z)) \rightarrow \perp$	(from 2,16)

We should note that we arrived at this valid reductio argument by a rough interpretation in which we took Anselm's "x is rather through z than through y" as "x is through z but not through y" and in which we simply stipulated the exclusivity principle. We will take up these points again later.

3.2 *Case b): being each through itself*

In case b), Anselm argues in a way that resembles a short version of an argument from M1, in which Anselm wanted to show that all that is called good is called so only because of something all good things have in common.

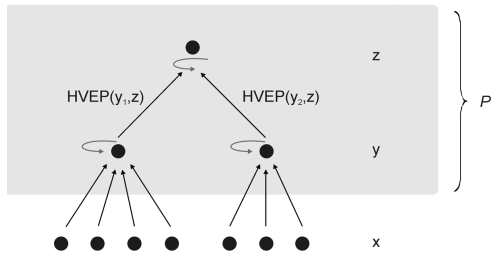
(S9) If, however, these exist separately, each through itself, there is, at any rate, some power or property of existing through oneself, by which they are able to exist each through itself.

(S10) But, there can be no doubt that, in that case, they exist through this very power, which is one, and through which they are able to exist, each through itself.

(S11) More truly, then, do all things exist through this very being, which is one, than through these, which are more than one, which, without this one, cannot exist.

Therefore, if y_1 and y_2 both have the property of being through itself, then, says Anselm, there must be a certain z such that the y's have this property or nature (*natura*) of being-through-itself from z. Hence, we face a new relation here, namely, of "having the power to be through oneself from", which comes down formally to a two-place predicate HVEP (for Latin "… *habet vis existendi per se per* …").

Case b)



Using this relation HVEP, Anselm argues as follows:

(S9a) "If, however, these [= the "*plura*" from the right disjunct] exist separately, each through itself", i. e., for our y_1 and y_2 with $Py_1 \wedge Py_2 \wedge y_1 \neq y_2$ holds:
$EP(y_1,y_1) \wedge EP(y_2,y_2)$

(S9b) "there is, at any rate, some power or property of existing through oneself (*existendi per se*), by which they are able to exist each through itself", i. e., there is a z from which all y that are through themselves have the power to be through themselves:
$\exists z\, \forall y\, (EP(y,y) \rightarrow HVEP(y,z))$.
Call this metaphysical assumption (HVEP1).

(S10) "But, there can be no doubt that, in that case, they exist through this very power, which is one, and through which they are able to exist, each through itself", i. e., what has the power to exist through itself from z, that is also through z:
$\forall y,z\, (HVEP(y,z) \rightarrow EP(y,z))$.
Call this metaphysical assumption (HVEP2).

(S11) "More truly, then, do all things exist through this very being, which is one, than through these, which are more than one, which, without this one, cannot exist." This resembles steps (S8b) and (S8c) in case a). Once again, we roughly take "more truly A than B" to mean "A and not B". Hence,

(S11a) "All things exist through this one being"; hence, all x_1 and x_2 that have their existence through y_1 and y_2, are in fact through z:
$\exists z\, (EP(x_1,z) \wedge EP(x_2,z))$ and

(S11b) this is "more truly" the case than that they are through y_1 and y_2. Hence, the x's are not through the y's:
$\neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$.

Hence, we arrive at the following formal reconstruction of case b):

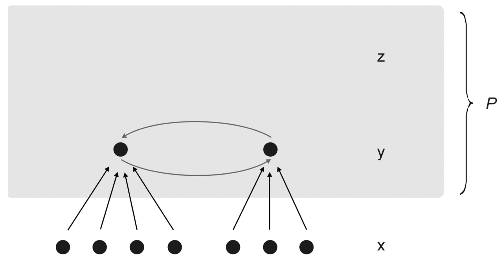
1		$\exists y_1,y_2\, (Py_1 \wedge Py_2 \wedge y_1 \neq y_2)$	(right disjunct of S6)
2	(S9a)	$EP(y_1,y_1) \wedge EP(y_2,y_2)$	(case assumption b)
3	(S9b)	$\exists z\, \forall y\, (EP(y,y) \rightarrow HVEP(y,z))$	(HVEP1)
4		$\exists z\, (HVEP(y_1,z) \wedge HVEP(y_2,z))$	(from 2,3)
5	(S10)	$\forall y,z\, (HVEP(y,z) \rightarrow EP(y,z))$	(HVEP2)
6		$\exists z\, (EP(y_1,z) \wedge EP(y_2,z))$	(from 4,5)
…	(S11)	[continued as in case a)]	

Line 6 corresponds to the assumption of case a); hence, the formal reconstruction can be continued as in that case.

3.3 *Case c): being mutually through each other*

Finally, in case c), we have two y's (y_1 and y_2), which are mutually through each other.

Case c)



In this case, Anselm argues as follows:

(S12) But that these beings exist mutually through one another, no reason can admit; because it is an irrational conception that anything should exist through a being on which it confers existence.

(S13) For not even beings of a relative nature exist thus mutually, the one through the other.

(S14) For, though the terms master and servant are used with mutual reference, and the men thus designated are mentioned as having mutual relations, yet they do not at all exist mutually, the one through the other, because these relations exist through the subjects to which they are referred.

Mutually-being-through-one-another is generally impossible (S12). This is the case for relative things (S13) and hence, *a fortiori,* for non-relative things such as existence providers apparently are. The claim about relative things (S13) is illustrated in (S14) by the example of master and servant, which is quite similar to P2, where the difference between being in the intellect and being in reality is illustrated by the example of a painter who initially has a painting only in his mind before he paints it. Here, Anselm illustrates that two relations – or two relata according to these relations – need not exist through one another, even if the relations hang as closely together as a relation (being-the-master-of) and its converse (being-the-servant-of).

The illustrative argument for the antisymmetry of EP that contains the *a fortiori* inference is not reconstructed in detail because its status seems to be more that of a justification for an assumption than a logically perspicuous chain of inferences. Therefore, the reconstruction is restricted to the following thoughts from (S12):

(S12a) "However, that these beings exist mutually through one another", that is, that there are x and y such that
$x \neq y \wedge EP(x,y) \wedge EP(y,x)$,

(S12b) "no reason can admit; because it is an irrational conception that …", that is, (S12a) is false because it is also false that

(S12c) "anything should exist through a being on which it confers existence", that is, because EP is antisymmetric:
$x \neq y \wedge EP(x,y) \rightarrow \neg EP(y,x)$ for arbitrary x, y.

Because we have left the plausibility argumentation out of the account, we obtain the following short derivation of a contradiction:

1		$\exists y_1,y_2\,(Py_1 \wedge Py_2 \wedge y_1 \neq y_2)$	(right disjunct of S6)
2		$EP(y_1,y_2) \wedge EP(y_2,y_1)$	(case assumption c)
3		$\exists x,y\,(x \neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 1,2)
4	(S12c)	$\forall x,y\,(x \neq y \wedge EP(x,y) \rightarrow \neg EP(y,x))$	(Antisym(EP))
5	(S12a)	$\neg\exists x,y\,(x \neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 4)
6		$\bot$	(from 3,5)

4. Preliminary reconstruction

Putting together the partial reconstructions of the several parts of the argument, we arrive at the following reconstruction:

0		$\forall y\,(Py :\leftrightarrow \exists x\, EP(x,y))$	(def(P))
1	(S6)	$\forall y_1,y_2\,(Py_1 \wedge Py_2 \rightarrow y_1=y_2) \vee \exists y_1,y_2\,(Py_1 \wedge Py_2 \wedge y_1 \neq y_2)$	(premise)
2	(S7)	$\exists y_1,y_2\,(Py_1 \wedge Py_2 \wedge y_1 \neq y_2) \rightarrow \exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1))$	(premise)
3		$\exists y_1,y_2\,(Py_1 \wedge Py_2 \wedge y_1 \neq y_2)$	(assumption)
4	(S8a)	$\exists z\,(EP(y_1,z) \wedge EP(y_2,z))$	(case assumption a)
5		$EP(x_1,y_1) \wedge EP(x_2,y_2)$	(from 0,3)
6		$EP(y_1,z) \wedge EP(y_2,z)$	(from 4)
7		$\forall x,y,z\,(EP(x,y) \wedge EP(y,z) \rightarrow EP(x,z))$	(Trans(EP))
8	(S8c)	$EP(x_1,z) \wedge EP(x_2,z)$	(from 5,6,7)
9		$\forall x,y,z\,(EP(y,z) \wedge EP(x,z) \wedge y \neq z \rightarrow \neg EP(x,y))$	(Excl)
10		$EP(y_1,z) \wedge EP(x_1,z)$	(from 6,8)
11		$y_1 \neq z \rightarrow \neg EP(x_1,y_1)$	(from 9,10)
12		$EP(y_2,z) \wedge EP(x_2,z)$	(from 6,8)
13		$y_2 \neq z \rightarrow \neg EP(x_2,y_2)$	(from 9,12)
14		$y_1 \neq z \vee y_2 \neq z \rightarrow \neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 11,13)
15		$y_1 \neq y_2 \rightarrow y_1 \neq z \vee y_2 \neq z$	(inequality axioms)
16		$y_1 \neq y_2 \rightarrow \neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 14,15)
17	(S8b)	$\neg(EP(x_1,y_1) \wedge EP(x_2,y_2))$	(from 3,16)
18		$\bot$	(from 5,17)
19		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \rightarrow \bot$	(from 4,18)
20	(S9a)	$EP(y_1,y_1) \wedge EP(y_2,y_2)$	(case assumption b)
21	(S9b)	$\exists z\,\forall y\,(EP(y,y) \rightarrow HVEP(y,z))$	(HVEP1)
22		$\exists z\,(HVEP(y_1,z) \wedge HVEP(y_2,z))$	(from 20,21)
23	(S10)	$\forall y,z\,(HVEP(y,z) \rightarrow EP(y,z))$	(HVEP2)
24		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z))$	(from 22,23)

25	(S11)	$\bot$	(from 19,24)
26		$EP(y_1,y_1) \wedge EP(y_2,y_2) \rightarrow \bot$	(from 20,25)
27		$EP(y_1,y_2) \wedge EP(y_2,y_1)$	(case assumption c)
28		$\exists x,y\,(x \neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 3,27)
29	(S12c)	$\forall x,y\,(x \neq y \wedge EP(x,y) \rightarrow \neg EP(y,x))$	(antisymm(EP))
30	(S12a)	$\neg\exists x,y\,(x \neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 29)
31		$\bot$	(from 28,30)
32		$EP(y_1,y_2) \wedge EP(y_2,y_1) \rightarrow \bot$	(from 27,31)
33		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1)) \rightarrow \bot$	(from 19,26,32)
34		$\exists y_1,y_2\,(Py_1 \wedge Py_2 \wedge y_1 \neq y_2) \rightarrow \bot$	(from 2,33)
35		$\forall y_1,y_2\,(Py_1 \wedge Py_2 \rightarrow y_1=y_2)$	(from 1,34)
36	(S5)	$\forall x\,\exists y\,(Ex \rightarrow EP(x,y))$	(conclusion 1^{st} part)
37	(S15)	$\exists! y\,\forall x\,(Ex \rightarrow EP(x,y))$	(from 0,35,36, and additional assumpt.)

In this version, the *reconstruens* is not yet a valid argument because several steps that lead to 37 have been omitted. However, because some other weaknesses will demand that we rework the *reconstruens*, we will put these issues aside until we have reached a solution of the weaknesses.

5. Kernel of the argument II: completeness of the case distinction

In all three cases a)–c), we have found valid derivations of contradictions using only explicit premises of Anselm's as well as principles to which, one can reasonably suppose, Anselm would have subscribed. The decisive question now is whether the case distinction a)–c) is complete, i. e. whether the premise taken from (S7) is true (line 2 in the preliminary reconstruction). However, this seems obviously not to be the case. In addition to the cases that a) two y's can be reduced to a common z, that b) two of the y's are EP-reflexive, and that c) there are EP loops of two y's, there are further possibilities: there could be d) EP loops of length greater than 2 or e) infinite paths through the field of the relation EP.

Case d) could, however, easily be reduced to the case of an EP loop of length 2 by the transitivity of EP: a finite circle of n objects y_i such that $EP(y_1,y_2)$, $EP(y_2,y_3)$, …, $EP(y_{n-1},y_n)$, $EP(y_n,y_1)$ would finally result in $EP(y_1,y_n)$ and $EP(y_n,y_1)$ by $n-2$ applications of transitivity. This situation of an EP loop of length 2 was excluded in case c). Hence, the matter of fact could be easily resolved in this case, and only an interpretative question would remain of why Anselm has not considered this case.

The situation in case e) is completely different. It may be the case that there are infinite chains among the y's, i. e., a series $(y_i)_{i=1,2,3,\ldots}$ of elements from the field of EP, such that for all natural numbers i, $EP(y_i,y_{i+1})$, and for all $j<i$, $y_i \neq y_j$ holds (i. e., the chain is not a large circle, as in case d)). This case, too, would be a case in which each element is "through a different one" without this chain ever ending or becoming circular. Anselm gives no hint of how to address this case.

Several reasons are conceivable for case e) to be impossible:

1. One presupposes that the world under consideration contains only finitely many objects. In that case, there can be no infinite chains of elements. However, the question is whether an argument on the uniqueness of God should be conceived as depending on such a bold metaphysical claim about the world and whether Anselm did so. The presupposition of a quantitatively finite world is conceivable, but then it would be difficult to explain why an excellent logician such as Anselm would remain silent about such a strong premise.
2. One presupposes a regress exclusion principle with respect to EP, i. e., a principle to the effect that no infinite chains of things where one is through the next one in the chain are possible. Such principles of regress exclusion, however, are not unproblematic. They cannot be taken as "obvious". Thus, for example, Thomas Aquinas exercises care in differentiating between admissible and inadmissible infinite regresses when, in the *Summa contra gentiles* I,13, he discusses the extended version of his famous proof "*ex motu*" from *Summa theologiae* I,2,3. Hence, in this case, one would ascribe a strong metaphysical premise to Anselm, leaving it unclear why Anselm has not mentioned it.
3. One introduces a new relation EP^+ of "being ultimately through something" and postulates that whatever exists through something must also have an ultimate source of its being: $\forall x,y\ (EP(x,y) \rightarrow \exists z\ EP^+(x,z))$. Formally speaking, this comes down to the stipulation that EP chains must have last elements.

This stipulation is weaker than the regress exclusion (alternative 2) because if there are no regresses (and no circles), each chain will reach a last element after finitely many steps. Conversely, however, it is not the case that the existence of last elements excludes infinite regresses. For example, consider the set of 1 and the fractions (n-1)/n for natural numbers n>0 with the usual ordering from the rational numbers, that is, the set of the fractions 1/2, 2/3, 3/4, 4/5, 5/6, … which converge to 1, plus the number 1 itself; then, 1 is a last element in this set, but 1 has infinitely many predecessors such that infinite regresses w.r.t. the >-relation are well possible.

The introduction of EP^+ is by no means "*ad hoc*". We found two places in Anselm's text that might be read as hints toward EP^+. The first place was in case a), when we had an x existing through y and y existing through z, and Anselm said that x was "rather" (*potius*) through z than through y (S8). The second place was in case b), when Anselm said that the x's are "more truly" (*verius*) through the z by which the y's had the power to be each through itself (S11). Provisionally, we decided to read these text passages using only EP, paying the price for this sparing interpretation by having to read "F rather/more truly than G" as "$F \wedge \neg G$". Because we have not arrived at a horizontally intact derivation based on this decision, we should now revise our decision and try the introduction of EP^+ instead.

6. Alternative reconstruction

In this section, I first put together our results and problems, then provide an overview of the most important properties of EP and EP^+, and finally suggest an alternative reconstruction of the whole argument. In our attempt to reconstruct Anselm's argument, we faced two lasting difficulties:

1. In tackling the sub-cases, it was occasionally difficult to see where exactly Anselm saw the contradiction. We were only able to solve this problem temporarily by introducing additional assumptions such as (Excl), which are problematic in the context of an Augustinian metaphysics such as Anselm's.
2. The case distinction itself turned out to be obviously incomplete. The case of larger loops was immediately resolvable, whereas the other case of infinite regresses was not.

In our discussion of the regress problem, we came up with the idea of introducing an additional relation EP^+, which has some support from Anselm's own statements ("*potius*" in S8, "*verius*" in S11). Using EP^+, we might obtain a valid argument in the following way.

We have the relation EP(x,y) with the intended meaning of "x exists through y" (<u>*e*</u>*st* <u>*p*</u>*er*). We take EP to have the following properties:

(EP1)	$\forall x,y\ (EP(x,y) \rightarrow Ex \wedge Ey)$	[EP only on existing things]
(EP2)	$\forall x,y,z\ (EP(x,y) \wedge EP(y,z) \rightarrow EP(x,z))$	[Transitivity]
(EP3)	$\forall x,y\ (EP(x,y) \wedge x \neq y \rightarrow \neg EP(y,x))$	[Antisymmetry]
(EP4)	$\forall x\ \exists y\ (Ex \rightarrow EP(x,y))$	[Left-side totality on E – "Whatever exists, exists through something" – intermediate conclusion of the first part of the argument]

In addition to EP, we now consider $EP^+(x,y)$ with the intended meaning "x is *ultimately* through y". EP^+ can be defined as follows:

$$EP^+(x,y) :\leftrightarrow EP(x,y) \wedge \neg\exists z\ (z \neq y \wedge EP(y,z)).$$

Explanation: If something, x, exists ultimately through y, then y cannot in turn exist through some other z (for then x would ultimately exist through z rather than through y). From the definition of EP^+ and the transitivity of EP, it follows immediately that EP^+ extends EP transitively to the right: $EP(x,y) \wedge EP^+(y,z) \rightarrow EP^+(x,z)$. In sum, EP^+ has the following properties, among which EP^+1 is the definition, EP^+2 is an immediate implication of the definition of EP^+ and the transitivity of EP, and EP^+3 is a metaphysical assumption:

(EP^+1)	$\forall x,y\ (EP^+(x,y) :\leftrightarrow EP(x,y) \wedge \neg\exists z\ (z \neq y \wedge EP(y,z)))$
(EP^+2)	$\forall x,y,z\ (EP(x,y) \wedge EP^+(y,z) \rightarrow EP^+(x,z))$
(EP^+3)	$\forall x,y\ (EP(x,y) \rightarrow \exists z\ EP^+(x,z))$.

For the sake of simplicity, we introduce a short-hand notation for the "more than one" (<u>*p*</u>*lura*) through which exists whatever exists (in the distributive sense according to which for whatever exists, there is one among the "(possibly) more than one" through which it is):

(P^+) $\quad P^+y :\leftrightarrow \exists x\ EP^+(x,y)$.

A handy way of talking about P^+y is to say that y is an ultimate source of being.

The goal of the proof is to show that there is one and only one ultimate source of being. To achieve this goal, one must show that there is an ultimate source of being and that two ultimate sources of being must be identical. For that goal, we do not need to amplify the intermediate conclusion from (S5) because from $\forall x\ (Ex \rightarrow \exists y\ EP(x,y))$ we can already infer $\forall x\ (Ex \rightarrow \exists y\ EP^+(x,y))$ by (EP^+3).

In sum, we arrive at the following reconstruction of (the second part of) the argument from M3:

1		$\forall y\,(P^{+}y :\leftrightarrow \exists x\, EP^{+}(x,y))$	(def(P^{+}))
2		$\forall x,y\,(EP^{+}(x,y) :\leftrightarrow EP(x,y) \wedge \neg\exists z\,(z\neq y \wedge EP(y,z)))$	($EP^{+}1$)
3	($S6_D$)	$\forall y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \rightarrow y_1=y_2) \vee \exists y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \wedge y_1\neq y_2)$	(premise)
4	(S7)	$\exists y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \wedge y_1\neq y_2) \rightarrow \exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1))$	(premise)
5		$\exists y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \wedge y_1\neq y_2)$	(assumption)
6	(S8a)	$\exists z\,(EP(y_1,z) \wedge EP(y_2,z))$	(case assumption a)
7		$EP^{+}(x_1,y_1) \wedge EP^{+}(x_2,y_2)$	(from 1,5)
8		$EP(x_1,y_1) \wedge EP(x_2,y_2)$	(from 2,7)
9		$EP(y_1,\underline{z}) \wedge EP(y_2,\underline{z})$	(from 6)
10		$\forall x,y,z\,(EP(x,y) \wedge EP(y,z) \rightarrow EP(x,z))$	(EP2)
11	(S8c)	$EP(x_1,\underline{z}) \wedge EP(x_2,\underline{z})$	(from 8,9,10)
12		$\forall x,y,z\,(EP(x,y) \wedge EP(y,z) \wedge z\neq y \rightarrow \neg EP^{+}(x,y))$	(from 2)
13		$\underline{z}\neq y_1 \rightarrow \neg EP^{+}(x_1,y_1)$	(from 8,9,12)
14		$\underline{z}\neq y_2 \rightarrow \neg EP^{+}(x_2,y_2)$	(from 8,9,12))
15		$\underline{z}\neq y_1 \vee \underline{z}\neq y_2 \rightarrow \neg(EP^{+}(x_1,y_1) \wedge EP^{+}(x_2,y_2))$	(from 13,14)
16		$y_1\neq y_2 \rightarrow \underline{z}\neq y_1 \vee \underline{z}\neq y_2$	(inequality axiom)
17		$y_1\neq y_2 \rightarrow \neg(EP^{+}(x_1,y_1) \wedge EP^{+}(x_2,y_2))$	(from 15,16)
18	(S8b)	$\neg(EP^{+}(x_1,y_1) \wedge EP^{+}(x_2,y_2))$	(from 5,17)
19		$\perp$	(from 7,18)
20		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \rightarrow \perp$	(from 6,19)
21	(S9a)	$EP(y_1,y_1) \wedge EP(y_2,y_2)$	(case assumption b)
22	(S9b)	$\exists z\,\forall y\,(EP(y,y) \rightarrow HVEP(y,z))$	(HVEP1)
23		$\exists z\,(HVEP(y_1,z) \wedge HVEP(y_2,z))$	(from 21,22)
24	(S10)	$\forall y,z\,(HVEP(y,z) \rightarrow EP(y,z))$	(HVEP2)
25		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z))$	(from 23,24)
26	(S11)	$\perp$	(from 20,25)
27		$EP(y_1,y_1) \wedge EP(y_2,y_2) \rightarrow \perp$	(from 21,26)
28		$EP(y_1,y_2) \wedge EP(y_2,y_1)$	(case assumption c)
29		$\exists x,y\,(x\neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 5,28)
30	(S12c)	$\forall x,y\,(x\neq y \wedge EP(x,y) \rightarrow \neg EP(y,x))$	(EP3)
31	(S12a)	$\neg\exists x,y\,(x\neq y \wedge EP(x,y) \wedge EP(y,x))$	(from 30)
32		$\perp$	(from 29,31)
33		$EP(y_1,y_2) \wedge EP(y_2,y_1) \rightarrow \perp$	(from 28,32)
34		$\exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1)) \rightarrow \perp$	(from 20,27,33)
35		$\exists y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \wedge y_1\neq y_2) \rightarrow \perp$	(from 4,34)
36		$\forall y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \rightarrow y_1=y_2)$	(from 3,35)
37	(S5)	$\forall x\,\exists y\,(Ex \rightarrow EP(x,y))$	(EP4)
38		$\forall x,y\,(EP(x,y) \rightarrow \exists z\, EP^{+}(x,z))$	($EP^{+}3$)
39		$\forall x\,(Ex \rightarrow \exists z\, EP^{+}(x,z))$	(from 37,38)
40		$\exists x\, Ex$	(premise)
41		$E\underline{x}$	(from 40)
42		$\exists z\, EP^{+}(\underline{x},z)$	(from 39,41)
43		$EP^{+}(\underline{x},\underline{z})$	(from 42)
44		$\forall x,z\,(EP^{+}(x,z) \rightarrow EP^{+}(x,\underline{z}))$	(from 1,36,43)
45		$\forall x\,(Ex \rightarrow EP^{+}(x,\underline{z}))$	(from 39,44)
46		$\exists y\,\forall x\,(Ex \rightarrow EP^{+}(x,y))$	(from 45)
47		$\forall x\,(Ex \rightarrow EP^{+}(x,y_1)) \wedge \forall x\,(Ex \rightarrow EP^{+}(x,y_2))$	(assumption)
48		$P^{+}y_1 \wedge P^{+}y_2$	(from 1,40,47)
49		$y_1=y_2$	(from 36,48)
50		$\forall y_1,y_2\,(\forall x\,(Ex\rightarrow EP^{+}(x,y_1)) \wedge \forall x\,(Ex\rightarrow EP^{+}(x,y_2)) \rightarrow y_1=y_2)$	(from 47,49)
51	(S15)	$\exists! y\,\forall x\,(Ex \rightarrow EP^{+}(x,y))$	(from 46,50)

Using EP^{+}, we finally obtain a valid argument that has improved in two respects. First, the dubious reason (Excl) is replaced by a property of EP^{+} that follows immediately from its definition. Second, the case distinction in (S7) can now be assumed to be complete; that is, the second premise of the *reconstruens*

(P4) $\exists y_1,y_2\,(P^{+}y_1 \wedge P^{+}y_2 \wedge y_1\neq y_2) \rightarrow \exists z\,(EP(y_1,z) \wedge EP(y_2,z)) \vee (EP(y_1,y_1) \wedge EP(y_2,y_2)) \vee (EP(y_1,y_2) \wedge EP(y_2,y_1))$

now probably follows from what can be imagined as a formalized Anselmian metaphysics. That y_1 and y_2 are presupposed as *ultimate* sources of being would now help to exclude the original case e) of infinite series.[11] However, both excluding the original cases d) and e) explicitly, or showing that (P4) holds, would require many more steps in the proof and more substantial formal tools than the ones used in our *reconstruens*. For example, we would need tools that enable us to talk about EP loops with variable length. As this would exceed the reasonable restrictions on the length of this paper and on the focus of what can count as a reconstruction of the given argument, I decided not to proceed any further in this direction in this paper.

7. Conclusions

We have logically analyzed an argument from Anselm's M3 to the effect that there is one and only one source of being. In our first reconstruction, we used only the relation EP and faced two problems: that we had to assume a problematic metaphysical premise (Excl) and that Anselm's case distinction was obviously incomplete. In our second attempt, we made use of another relation, EP^{+}, and obtained a *reconstruens* that is valid, avoids the problematic premise (Excl), and solves the issue of the incomplete case distinction. What about the premises used in this *reconstruens*?

[11] This is similar to the concept of a *causa sui* but need not be identical to it because "*esse per*", in Anselm's view, is less reflexivity-phobic than the concept of a cause.

All assumptions in the proof are subsequently bound. Hence, only the following 12 premises remain open (line numbers in the reconstruction): 1–4, 10, 16, 22, 24, 30, 37, 38 and 40. Two of these (1, 2) are definitions. Two others (3, 16) are logical truths or basic axioms of (in)equality. Another pair (10, 30) concern two obvious formal properties of EP: transitivity and antisymmetry. Hence, we have six contentual premises left over for discussion: 4, 22, 24, 37, 38 and 40.

Premise 4 is the completeness of the case distinction, which has now become unproblematic because we adapted the formalism to that end. Premises 22 and 24 are assumptions that are deeply rooted in an Anselmian metaphysics: if two things have the same nature, there is one common thing from which they have this nature (HVEP1, prem. 22), and they exist by virtue of the thing through which each has the nature to be through itself (HVEP2, prem. 24). Premise 37 was the conclusion of the first part of the argument, which we decided not to reconstruct in detail. However, we have discussed Anselm's reasons for this assumption above. Suffice it to say here that premise 37 carries most of the "existential weight" of the proof. The remaining portion of the "existential weight" is carried by the additional assumption (EP^+3, prem. 38), which we added to avoid the problem of how to exclude the case of an infinite regress of beings. As noted above, this is a move that requires considerable additional metaphysical commitments, as I will discuss below. Finally, premise 40 only says that something exists; Anselm would certainly have subscribed to it. The only thing to be asked here is the intriguing question of whether this premise is empirical and makes Anselm's argument an *a posteriori* proof.

Hence, lines 37 and 38 in the reconstruction are the most unsecured premises from the point of view of a critical sympathetic to Anselm's metaphysics. One may combine the two lines as in line 39:

(39) $\forall x\ (Ex \rightarrow \exists z\ EP^+(x,z))$,

and ask how one could one argue in favor of such a premise.

Each existing x exists through a y. Such a y again exists through a z, the z through w, and so on. In this way, we obtain chains of existential causes. For such chains, in general, there are three possibilities:

(i) they are circular,
(ii) they are non-circular and finite, or
(iii) they are non-circular and infinite.

Circularity (i) is excluded by Anselm's case c) in connection with the transitivity of EP. Finite linear chains (ii) are the case Anselm can settle by his argument. In that case, the elements earlier in the chain are *rather* through the elements much later in the chain than through elements only slightly later in the chain (also using transitivity). If linear chains of elements x_i such that $EP(x_0,x_1)$, $EP(x_1,x_2)$, $EP(x_2,x_3)$, … end with x_n, that is, there is no $y \neq x_n$ such that $EP(x_n,y)$, then this y is the last element in question, i. e., $EP^+(x_i,y)$ holds for all $i=0,\ldots,n$.

Finally, the possibility of infinite regresses (iii) cannot be excluded in general. This point, however, touches upon deep metaphysical and logical problems that go far beyond the limited focus of this paper. Generally speaking, most medieval authors viewed some important metaphysical relations as excluding infinite regresses. However, the more careful authors have made considerable efforts to argue that such a view is justified. To indicate the logical intricacy, it may suffice to say that one must discern between an infinite regress and the non-existence of first/last elements (in our case, a y such that $EP^+(x,y)$). Even after an infinite number of elements x_i such that $EP(x_0,x_1)$, $EP(x_1,x_2)$, $EP(x_2,x_3)$, … there can be a y such that $EP^+(x_i,y)$ for all i. If there is no such y, then there surely is an infinite regress, but not the other way around (regresses do not exclude first/last elements).

In attempting to logically reconstruct Anselm's argument from M3, we first tried a more sparing interpretation that allowed us to arrive at a valid argument only by stipulating at least two doubtful principles. Then, by using the relation EP^+, which has no explicit counterpart in Anselm's text, we developed an alternative reconstruction. This reconstruction was motivated by two explicit Anselmian hints, and it potentially is extendable into a valid derivation whose premises are well grounded in Anselm's metaphysics. It seems that this is a case of the general interpretive need to carefully weigh closeness to the *reconstruendum* against the availability of reasons for the premises of the *reconstruens*.[12]

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