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Infinity in Aquinas' Doctrine of God

Christian Tapp

Ruhr University Bochum

1 Introduction

Thomas Aquinas uses the expression "infinite" and its derivatives in a variety of senses that range from the essential infinity of God himself to the quantitative infinity of the series of natural numbers or of the measure of an infinitely extended body (which he, as an Aristotelian, believes to be impossible). The chief aims of this paper are to present an overview of the various senses in which Aquinas uses "infinite" in his doctrine of God, to explain more precisely what these senses are and how they might be related, and to delineate the extent to which Aristotle's influential bans on (the realization of several senses of) infinity interfere with these senses.

I will begin by considering an apparent incompatibility between Aristotle's verdict on actual infinity and Aquinas' conception of God that motivates a somewhat more detailed account of Aristotle's bans (2.). Then, I will examine in which sense Aquinas calls God himself "infinite" (3.) and identify the epistemic implications of God's infinity (4.). Next, I will return to Aristotle's bans and discuss the tensions from which they suffer both in themselves and when confronted with Christian doctrine: quantitative infinity is discussed with regard to infinite regress arguments that exclude certain types of infinite series (5.); discrete quantitative infinity is discussed with regard to the Christian belief in resurrection (6.); and continuous quantitative infinity is discussed with regard to cosmology and the Christian belief in an omnipotent creator God (7.). Finally, a link between essential and quantitative infinity is noted (8.).

2 Basic Distinctions & Aristotle's Bans on Infinity

For a faithful Aristotelian such as Thomas Aquinas, there is some immediate tension in teaching that God is infinite. Because God is pure actuality (*actus purus*), he appears to be something that is actual and infinite. However, Aristotle has proven that actual infinity is impossible. Is it, hence, impossible that God exists?

One can block such an argument in two obvious ways: by disclaiming Aristotle's arguments regarding the impossibility of actual infinity or by denying that "being actually infinite" is used in the same sense in Aristotle's bans and in Aquinas' calling God "actual" and "infinite". Rather than discussing the soundness of Aristotle's arguments, I will focus on the second option in this paper.

A third option, namely, that Aquinas simply did not know about Aristotle's arguments against infinity, is irrelevant. Aquinas, who knew and even esteemed Aristotle's arguments, was much more restrictive in accepting quantitative infinity than, for example, Avicenna or Algazel.[1] In fact, from his distinctive Christian standpoint, he developed further arguments against the infinite in nature, such as an argument that proceeds from the biblical Book

[1] Aquinas compares his view with Avicenna's and Algazel's in, e.g., *STh* I,7,4 [= *Summa Theologiae*, p.I, q.7, a.4].

of Wisdom's saying: "You have ordered all things in measure, number, and weight" (Wisdom 11:20).[2] If something were quantitatively infinite, or so the argument goes, this infinite thing also would have to be ordered by measure and number. However, there are no infinite numbers. Hence, the infinite thing could not be ordered by number, and therefore, such an infinite thing is impossible. Such an argument seems valid with *plausible* premises as long as there is no consistent theory of infinite numbers. However, once such a theory is given, e.g., by Cantorian set theory, infinite sets are measurable by transfinite numbers.[3]

To develop the second option, we first need a clear understanding of Aristotle's bans as developed most prominently in his *Physics*. A very basic Aristotelian distinction is made between potential and actual infinity.[4] Potential infinity is the type of infinity we find if something can be made increasingly large in a stepwise manner. For every given finite value, the process will outgrow the value at some point. Conversely, actual infinity consists of something that actually is infinitely large. An actually infinite value is so large that it *is*, not *becomes*, larger than all finite values. (Contemporary philosophers and logicians have raised many doubts regarding whether there is a precise sense of potential infinity. However, let us take this concept for granted to understand Aquinas' account of Aristotle.)[5]

The distinction between two types of quantitative infinity, namely, multitude and magnitude, features prominently in medieval accounts of Aristotle's impossibility claims. Magnitude is a continuous quantity that measures the size of something; stating a magnitude answers the question "How much ...?" Multitude is a discrete quantity that measures the number of some things of the same sort; stating a multitude answers the question "How many ...?"

With this terminology in mind, Aristotle's position regarding infinity in nature can be explained as follows (see the accompanying chart): actual infinities are completely impossible, including actually infinite magnitudes and

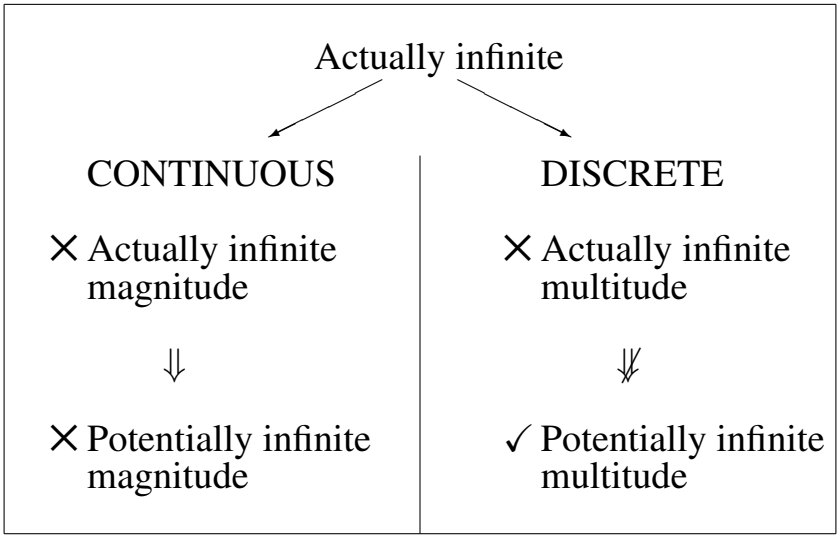
[2] See *STh* I,7,4 c.a.

[3] See [Tapp, 2011b].

[4] Aristotle, *Physics* III; for a general account of the potentiality/actuality distinction, see, e.g., *Metaphysics* IX,6.

[5] The doubts concern the question of whether the difference between potential and actual infinity could ever be made precise. Many philosophers suggest that any potentially infinite set presupposes an actually infinite one. Every precisely determinable mathematical entity such as a set, a class, or a number is either finite or infinite – *tertium non datur* (see, for example, [Niebergall, 2004], 174).

actually infinite multitudes. In the case of magnitudes, even potential infinity is excluded because, according to Aristotle, only existing magnitudes can be manipulated and the universe is of finite size. In the case of multitudes, however, potential infinity is admitted by Aristotle because it results from the infinite process of dividing a continuum, which is a process that cannot consistently be thought to reach an end after a finite number of steps.



Aristotle explicitly developed his position in the context of his account of nature. Hence, an easy initial way to block the atheistic argument is to point to the fact that, for Aquinas, God does not belong to the realm of nature, and therefore, Aristotle's arguments *need* not apply to him.[6] However, to see that the arguments *do* not apply, we can underscore the distinction between using "infinite" categorematically and syncategorematically.[7] In general, to use a predicate *syncategorematically* means to use it as a modifier for other predicates or functional expressions. For example, "infinite" is used to modify "time" to "infinite time", "goodness" to "infinite goodness", or "power" to "infinite power". In many cases, it is helpful to further distinguish between

[6]Aquinas himself frequently uses this strategy, for example, when discussing the range of the classical thesis "*ex nihilo nihil fit*": This is valid when one considers nature in itself but not if God is concerned, see *SCG* II,37 [= Thomas Aquinas, *Summa Contra Gentiles*, l.II, c.37].

[7]For the distinction of categorematical and syncategorematical use of predicates, see [Bocheński, 1970], 179–182; and [Kambartel, 1971], 33.

extensional and intensional modifications. To say that a beam is infinitely white means extensionally that the beam is white and infinitely long such that its whiteness extends over an infinite material body. However, being infinitely white can also mean intensionally that the intensity of whiteness is infinite. (Aquinas believed that both types of infinity were impossible in nature). "Infinite" can also be used "*categorematically*", namely, when "infinite" is not modifying the meaning of another predicate but has an intended meaning in itself.

When Aquinas calls God "essentially infinite", he uses "infinite" categorematically. (We will see later what "essential infinity" exactly means.) Although it seems that Aristotle's use of "infinite" is categorematical, it is not. As Aquinas cites Aristotle, "*infinitum quantitatem sequitur*" – "infinity belongs to the realm of the quantitative".[8] However, "infinity" taken in the quantitative sense is shorthand for "infinitely many" or "infinitely much". Hence, even if the syntax is categorematical because of the abbreviation, the quantitative infinity predicate is a case of syncategorematic use of "infinite". In Aquinas' view, Aristotle's bans concern the syncategorematical use of infinity in the quantitative sense and, therefore, do not directly affect Aquinas' teaching that God is categorematically and not quantitatively infinite.

3 Essential Infinity: The Creator as *Esse Subsistens*

What does Aquinas mean by calling God essentially infinite *simpliciter* using "infinite" categorematically without any qualification? Aquinas presents this doctrine most prominently in *quaestio* 7 of part I of the *Summa Theologiae*. He addresses this issue after presenting his arguments for God's existence and establishing the divine attributes of simplicity, perfection, and goodness.[9] After infinity has been addressed, the further attributes of omnipresence, immutability, eternity, and unity make the essentials of the doctrine of God as considered in himself complete.[10] (In other works, such as the *Summa Contra Gentiles*, the architecture is different.)

Of course, God cannot be called infinite in the sense in which the philoso-

[8] Thomas Aquinas, *SCG* I,43.

[9] *STh* I,3 (simplicity); I,4 (perfection); I,5–6 (goodness).

[10] *STh* I,8 (omnipresence); I,9 (immutability); I,10 (eternity); I,11 (unity).

phers of antiquity taught that the first principle of being is infinite. Many ancient philosophers of nature held that the first principle of being is material and hence imperfect. To be sure, Aquinas' conception of the Christian God was not that of an imperfect, material being. Therefore, why did Aquinas want to incorporate the doctrine of infinity in his system of thought at all? Which modifications have been necessary?

I believe that the infinity of the first principle was attractive for Aquinas because this concept was closely related to creation. From the first principle, the natural philosophers suggested that "things flow forth infinitely."[11] This suggestion fits well with Aquinas' Christian conception of God as the original creator and sustainer of the world. However, the identification with bodily matter needed to be dropped. Aquinas explains that matter and form, which are the two principles through which all concrete beings exist, are mutually limiting or terminating each other. In what sense do these principles limit each other? Form limits matter in so far as matter is pure potentiality; that is, it can be the matter of many different things. However, this is not the case if matter has received a form that makes it the matter of this particular thing. For example, a piece of clay can become a statue or a brick. If the clay is formed into a brick, it cannot become a statue anymore (at least not by the same type of manipulations as before.) In this way, form reduces the range of the possibilities of matter. Conversely, matter limits form in that a form considered in itself can be multiply instantiated, but if it is individualized or "materialized" by matter, it becomes the form of this or that particular thing. In this way, matter limits form by making it the form of a particular thing instead of many other similar things. Aquinas here speaks of a *contractio amplitudinis*, a contraction of range.

The two theoretical limiting cases of composition by form and matter are matter without form and form without matter. Both limiting cases can be said to be cases of infinity because the respective other element, and thereby its limiting function, is dropped. However, one of these cases, the case of matter without form, displays an imperfection because the form is what makes matter into a determined thing and provides it with actuality. Without form, one might say figuratively, matter would be an amorphous mass. Strictly speaking, prime matter (*materia prima*) is a pure abstraction in thought that has no reality in itself and would be nothing if it were to exist. In the other case,

[11] "*Res effluere in infinitum*", *STh* I,7,1.

form without matter or pure form, "infinite" has the sense of perfection because when a form is the form of this or that particular thing, it is contracted to this thing and made from something general into something particular. Hence, if this contraction does not occur, as in the case of pure form, we have a more perfect situation.

In this way, Aquinas delineates his own conception from those of the Presocratics. However, he also delineates his conception from the Philosopher and particularly from the concept of infinity in the *Physics*. Why so? Because, according to that concept, infinity is a quantitative predicate that applies to spatial things with a definite size. However, because God is not spatial and has no size, the quantitative predicate cannot apply to him. Note that Aquinas understands "quantitative" here as concerning spatial size or number; this is a narrower conception of quantitative than the conception he uses in the *Summa Contra Gentiles* (*SCG*).[12]

In the *SCG* account of infinity (*SCG* I,43), Aquinas uses "quantitative" in a much broader sense. He subscribes to Aristotle's ascription of "infinity" to the realm of quantity ("*infinitum quantitatem sequitur*") and distinguishes three or four senses of quantitative infinity as follows:

- *quantitas sec. multitudinem* (quantity in the sense of a multitude)
- *quantitas sec. quantitatem continuam*[13] (quantity in the sense of a continuous magnitude)
- *quantitas sec. magnitudinem spiritualem*[14] (quantity in the sense of a "spiritual" magnitude) in the two varieties
  - *quantum ad potentiam* (quantity with regard to power)
  - *quantum ad propriae naturae bonitatem sive completionem* (quantity with regard to the goodness or completion of one's own nature)

[12] As Norman Kretzmann has suggested, quantitative infinity as used in the *SCG* covers immaterial entities (see [Kretzmann, 1997], 166).

[13] The continuous quantity is also called "*magnitudo*" in STh and "*quantitas dimensiva*" later in SCG I,43.

[14] Kretzmann was correct that the term "spiritual magnitude" is misleading because there is nothing "spiritual" about intensive quantity (which is covered by "spiritual magnitude" as Aquinas uses this term) and because "spiritual" entities in the sense of immaterial ones can have a number or even be extended (see [Kretzmann, 1997], 166).

The first two types of quantity cannot apply to God. First, God cannot be infinite in the sense of an infinite multitude because of his uniqueness and simplicity: because he is unique (*SCG* I,42), it is impossible that there be several, not to mention infinitely many, Gods. Further, because he is simple, he cannot have infinitely many parts (*SCG* I,18). Second, God cannot be infinite in the sense of an infinite continuous quantity or magnitude because of his incorporality: being immaterial and having no body, God cannot have bodily or spatial parts (*SCG* I,20). Hence, God can be infinite only in the sense of a "spiritual magnitude" in one of the two senses: with regard to his power, i.e., the extension of his effects or the intension as proportionate to the "size" of the changes it can effect, or with regard to the completion of his own nature. However, Aquinas shows that the two versions are identical because having a certain power means being *in actu* in a certain respect, and that, in turn, means being completed in one's nature; in God's case, this nature is pure actuality. Hence, it is equivalent to say that God has infinite power and that God is completed actuality.

The importance of this teaching becomes immediately apparent when one considers its application to the doctrine of creation. Creating the world means bringing or keeping that which is "infinitely far from actuality and is not in any way in potentiality" into actuality. Because the power that an agent needs to bring about a state of affairs is proportional to the "remoteness" of the state of affairs from actuality, a creator *ex nihilo* needs infinite power.[15]

Aquinas notes that his task is to show that God is infinite in the sense of spiritual things, namely, in being completed or perfected in his nature. We should note that "infinite" as a negation of "finite" is usually used privatively: having limits is innate (*nata*) to created things such that if they do not have the limits they naturally have, they lack something. The case of God is different. Divine infinity does not mean the lack of something that God naturally should have but rather that his perfection has no limits:

> *nullus est perfectionis suae terminus sive finis, sed est summe perfectus. Et sic Deo infinitum attribui debet* – there is no terminus or limit to His perfection: He is supremely perfect. It is thus that the infinite ought to be attributed to God (SCG I,43).[16]

What we now have established is a conception of essential infinity, infin-

[15] *SCG* I,43. See [Kretzmann, 1997], 167–168.

[16] Translation http://dhspriory.org/thomas/ContraGentiles1.htm#43, 11.01.2014.

ity in essence, that can be applied to the Christian God. In fact, as Aquinas proceeds with his reasoning, this conception can be applied *only* to the Christian God. Other things may be infinite *secundum quid* (in a certain respect) but not *simpliciter*. The reason is that all creatures have received being, but God alone is his own subsistent being. The being of creatures is limited according to their natures, which are their determinations according to genus and species. Hence, their *esse* is limited in being received in a way that fits its determined nature. Even if there were infinitely many beings of infinitely many kinds that are infinitely extended, their infinity would be of a lesser type than God's infinity (*minoris infinitatis*) because they are beings of a certain kind. Hence, they are limited in at least one respect, namely, with regard to their being, which is received according to their nature as being something and therefore having an essence that is not identical to their existence.

In Aquinas' view, divine essential infinity implies many other syncategorematical attributions of infinity, e.g., having no limits with regard to space and time. God is ubiquitous or omnipresent in so far as he is extended over all space by virtue of his power of sustaining and his power of knowing.[17] Further, he is eternal in that he has no temporal limitations. In Aquinas' Boethian outlook, eternity means that God's duration extends over all of time although he is timeless in the sense of having no past or future and having his life "all at once".

4 Epistemic Implications of Essential (In)Finitude

According to the three largest monotheistic religions and classical Greek philosophy, God cannot be completely known. Further, God should not be completely known because he is a being who is much more venerable and worthy of adoration than anything in the world. Religious people conceive of God as a being of unsurpassable moral value, majesty, power, and knowledge. He is

[17] "*Post considerationem divinae perfectionis, considerandum est de eius infinitate, et de existentia eius in rebus, attribuitur enim Deo quod sit ubique et in omnibus rebus, inquantum est incircumscriptibilis et infinitus*" – "After considering the divine perfection we must consider the divine infinity, and God's existence in things: for God is everywhere, and in all things, inasmuch as He is boundless and infinite" (*STh* I,7,prooem).

not only a being "than which nothing greater can be conceived",[18] as Anselm of Canterbury famously suggested, but also a being "greater than what can be conceived".[19] Clearly, it is strongly in the interest of every religiously inclined thinker to acknowledge this supremacy of God.

For Aquinas, this concept resulted in a theoretical challenge because he was impressed by the capacity of human reason and intellect. Aquinas considered empirical and intellectual research to be fundamentally important and was convinced that mankind would proceed and extend its knowledge by engaging in that type of activity. However, if theological investigations make sense at all, will there not come a time when we finally understand God, make him completely tractable with our human epistemic instruments, or classify him based on the rest of our knowledge of the world?

In Aquinas' eyes, this idea would contradict God's supremacy and, perhaps even more importantly, his role in the ultimate destiny of all creation. The perfection of man, beatitude or being in heaven, consists in seeing God. This is the famous axiom of "*visio beatifica*", the seeing in which ultimate beatitude and fulfillment consist. How could anyone strive toward that destiny and how could it be an object of promise and transcendent hope if it were completely made an object of knowledge and mundane human activities? Here, we have reached an important challenge in the construction of a rational theology that combines both respect for the divine majesty and respect for the admirable capacity of human reason.

Aquinas' answer to this challenge assigns an important role to the concept of infinity. Roughly speaking, if complete knowledge of the divine being is an infinitely distant goal, then it is no problem that the finite amount of knowledge is increased indefinitely. In so far as this knowledge always remains finite, it will not completely embrace the infinite God. In this way, one can combine continuous human progress and essential divine inscrutability.

According to Aquinas' thinking, the situation is even more complicated because of his adherence to the doctrine that God is the simplest, most rational, and highest being. In fact, he is not *a* being but being *simpliciter*. According to medieval epistemology, this doctrine makes God in himself the most cognizable being. For our intellect, however, God is like the sun whose brightness hinders our visual cognition of itself, although it is, gen-

[18] Anselm of Canterbury, *Proslogion* II, passim.

[19] Anselm of Canterbury, *Proslogion* XV.

erally speaking, a prerequisite for all visual cognition.[20] Likewise, God's excessive light and being result in him exceeding our intellectual capacities and being incomprehensible to us (*quoad nos*), although he is the most comprehensible being as considered in himself (*per se*).

In his *Commentary* on the first chapter of the Gospel of John, Aquinas asserts his theory in a nutshell:

> Now a created intellectual substance is finite; hence it knows in a finite way. And since God is infinite in power and being, and as a consequence is infinitely knowable, he cannot be known by any created intellect to the degree that he is knowable. And thus he remains incomprehensible to every created intellect. 'Behold, God is great, exceeding our knowledge' (Job 36:26).[21]

Here, we face the opposite problem: if God's infinity implies that he is so great that he exceeds our knowledge in principle, how could a *visio beatifica* in the sense of seeing God as he is in himself be possible at all? In *STh* I,12,1, Aquinas poses this objection: does God's infinity not imply that he is completely unknowable? Aquinas' answer has several steps. The first step is taken in the article just quoted: God is infinite in the sense of a form without matter (see *STh* I,7,1), and in this sense, he is most knowable *in himself*. However, *for us* to know God would require us to receive the divine form, which is impossible for any created intellect (*STh* I,12,4) because God's being-in-the-knower can occur only according to the knower's mode of being, and this mode is finite *simpliciter*. Hence, Aquinas arrives at the theological doctrine he was aiming at: although the created intellect cannot come to know God by his natural capacities alone, it can know God if God gracefully unites himself with the created intellect (*STh* I,12,4).

What we have seen here is another function of the concept of infinity in Aquinas' doctrine of God. This concept helps bringing together God's essential incomprehensibility and supremacy with human progress in rational enquiry and with God's being comprehensible in himself to the highest possible degree.

[20] Aquinas makes frequent use of Aristotle's metaphor of the sun that cannot be seen by the birds of the night (*Metaphysics* II); see, for example, Aquinas' *Super Sententiis* l.1 d.17 q.1 a.4; *SCG* I,3, *SCG* III,45; *De veritate* q.8, a.3; or *STh* I,12,1: "*sicut sol, qui est maxime visibilis, videri non potest a vespertilione, propter excessum luminis*".

[21] *Super Io*. c.1 l.1, transl. http://dhspriory.org/thomas/John1.htm, 19.9.2013.

5 Discrete Quantitative Infinity I: Arguments by Infinite Regress Exclusion

Arguments by infinite regress exclusion (AIREs) can be found throughout Aquinas' works. An infinite regress argument is a particular type of *reductio*. Beginning from assumption A, one derives some sort of infinite series. Then, one adduces the premise that such an infinite series is impossible. Hence, one arrives at a contradiction from which one finally concludes that assumption A is false. AIREs are crucial in Aquinas' doctrine of God, and in turn, infinity is crucial for AIREs. Hence, AIREs must be discussed in a paper that examines infinity and Aquinas' doctrine of God.

5.1 An Example of an AIRE

The best known examples of AIREs are the first two of Aquinas' five ways of proving the existence of God.[22] The first argument consists of two parts. First, by appealing to Aristotelian physics, Aquinas proves that everything in motion must be moved by something else. Second, Aquinas uses the result of the first part to prove that there must be a first unmoved mover, which is God. Because the argument uses an AIRE in only its second part, I consider only the second part and take the result of the first part as an assumption. Aquinas' argument can be approximately summarized as follows:

(1) There is motion, i.e., there are things that are in motion.

(2) Everything in motion must be moved by something else.

(3) Ergo: There is (an initial segment of) a chain of moved things such that each item moves the next one.

(4) This chain cannot extend to infinity.

(5) Hence, there is a first element in the chain, a first mover.

(6) A first mover is God.

(7) God exists.

[22] See *STh* I,2,3.

This argument is problematic in several respects. For example, Aquinas proves thesis (2) by appealing to the Aristotelian doctrine of movement as an actualization of potencies. This makes the argument rely heavily on Aristotelian physics. Then, what is the status of premise (6)? Why must a first mover of the world be divine? What is the definition of God in the background, and how is it motivated? Further, what about uniqueness? Can there not be several "first" movers in the sense of beginning points of the chain? These are just some of the challenges posed by the argument. In the present context, my focus is exclusively on step (4): why is it impossible for the chain to extend to infinity?

Many commentators do not see a problem here. Some even claim explicitly that this premise is self-evident. Others say the premise follows immediately from the temporal and spatial finitude of the world. A third group takes the premise for granted because we finite human beings are generally incapable of surveying something infinite. Finally, Joseph Bocheński suggests in his formal reconstruction of the *prima via* that regress exclusion is an ontological premise that is not self-evident but that is in need of proof.[23]

We must first note that the exclusion of infinite regresses cannot be self-evident because some infinite regresses are possible. For example, Aquinas admits that when Jacob is the son of Isaac, who is the son of Abraham, etc., such a chain of generations can go to infinity without implying any contradiction. Aquinas makes this clear when he discusses arguments regarding the eternity of the world.[24] According to Aquinas, the Aristotelian doctrine that the world infinitely extends backwards in time cannot be refuted by reason alone. It is a matter of genuine faith that the world has a beginning and an end. Therefore, Aquinas doubts that infinite regresses reaching infinitely back into the past could generally be refuted.[25] Another example of an allowed regress

[23] [Bocheński, 1989], 246–247; see also the discussion in [Sobel, 2004], 181–190 (w.r.t. the analogous principle in the via secunda); for a not-easily-readable reconstruction of Aquinas' proof including his arguments for the regress exclusion principle, see [Salamucha, 1958].

[24] Aquinas' reason for allowing infinite regresses is the accidental nature of their ordering. He says that "it is accidental to Socrates' father that he is another man's son or not" (*SCG* II,38; translation from dhspriory.org, 25.4.2014).

[25] Thus, Aquinas would not readily subscribe to the premises of William Craig's so-called "Kalam cosmological argument" for the existence of God; see [Craig, 2000]; a collection of papers from a public debate between Craig and Smith can be found in [Craig and Smith, 1993].

is a sculptor or a blacksmith who requires a hammer for his work; each time a hammer breaks, he takes a new one, and thus, he approaches an infinite set of hammers if he works for an unlimited period of time.[26] This set of hammers is an example of a set of objects that is only accidentally infinite. According to Aquinas, if this regress were impossible, it would not be for conceptual reasons alone.

Hence, the regress in the *prima via* cannot be excluded by simply arguing that infinite regresses are impossible in general. We must ask instead what *types* of infinite regresses are impossible, and what are the *reasons* for their impossibility.

5.2 Reasons for Regress Exclusion

There are hundreds of examples of the regress exclusion principle in Aquinas' works but only a few instances in which Aquinas provides reasons for the principle. In the majority of the cases, he simply refers to Aristotle.[27]

For Aquinas, there are two types of regresses, possible and impossible ones. The chain of generations or the chain of hammers may well go to infinity. Such infinite regress series are called "accidentally infinite"; such a series just happens to grow to infinity without this being necessarily so.

Conversely, essentially ordered series of causes are excluded, such as when my soul moves my muscles, my muscles move my arm, my arm moves my hand, my hand moves the stick in my hand, and the stick moves a stone. In a series of that type, the movement of one element was only *in virtue of* its being moved by other things earlier in the series. Thus, the causal ordering is what Aquinas calls an "essential ordering". Conversely, Isaac's being begotten by Abraham is not the cause of Isaac begetting Jacob.

We can note the difference between accidentally and essentially ordered series of causes by refining the causality relation. Let us assume that causality is a four-place relation: a thing x's having property P is the cause of a thing y's having property Q. We can express this in symbols: C(x,P,y,Q). For the sake of simplicity, I omit temporal variables or indices from consideration. A (non-empty) series of things $(a_i)_{i \in I}$, whether finite or infinite, is called a "causal chain" iff

[26] *STh* I,7,4.

[27] For example, he mentions *Metaphysics* II; *Posterior Analytics* II,22; *Physics* III; *Physics* VIII; and even *De Generatione et Corruptione Animalium*.

(i) for all $i \in I$: if $i+1 \in I$, then there are properties P and Q such that C(a_i, P, a_{i+1}, Q) holds.

A causal chain $(a_i)_{i \in I}$ is *essentially ordered* if the following condition also holds

(ii) for all $i \in I$: if $i+2 \in I$, then there are properties P, Q and R such that C(a_i, P, a_{i+1}, Q) and C(a_{i+1}, Q, a_{i+2}, R) hold.

In essentially ordered chains, the caused and the causing properties of one and the same thing are the same.[28] Then, the regress exclusion principle amounts to the exclusion of infinite regresses in essentially ordered causal series. This captures the difference between the hand-stick-stone and the Abraham-Isaac-Jacob series. The stick's moving both is caused by the hand's moving and causes the stone's moving. However, it is not Isaac's being begotten by Abraham that causes Jacob to be begotten by Isaac.

Hence, premise 4 of the *prima via* is a meaningful assumption about the causal series of moved things. Because there is no general argument regarding the exclusion of essential infinite regresses, one must argue for its instantiations. Aquinas does so, for example, in the extended version of the *prima via* in *SCG* I,13. Two of his arguments are approximately as follows:

(1) If there were infinitely many moved things at the same time, they could be conceived of as a whole that is infinite but moved in finite time. However, an infinite whole being moved in finite time is impossible according to another proof of Aristotle.

(2) Movement would be impossible if there were no first source of movement. However, there is no first element in an infinite series. Hence, an infinite series of moved things is impossible.

Neither of these two arguments is completely convincing. For example, the second argument is problematic. At first, it is unclear whether we should actually subscribe to the premise that movement is possible only if there is a first source of movement. If this were Aquinas' thesis, there would be a much shorter proof for divine existence as follows: there is motion; motion would

[28] The idea to analyze the difference between essentially and accidentally ordered causal chains in these terms is taken from [Brown, 1966].

be impossible without a first source of it; and hence, there is a first source of motion. It would be unconvincing to assign to Aquinas a justification for a premise in a lengthier argument, which is so strong that it would imply his overall conclusion almost immediately.

However, even if we granted Aquinas this principle, the rest of the argument would still be logically defective. The problem is that the infinity of a series does not preclude it from having a first element.[29] Having no first element implies being extended backwards to infinity but not the other way around. Hence, even if we assume that without a first source there would be no movement at all, this only shows that we must assume a first source and not that an infinite series of moved things is impossible.

Because of a lack of space and time, I must reserve a more detailed analysis of Aristotle's and Aquinas' arguments regarding IR exclusion for another opportunity. However, I will mention two critical points here. First, if Aristotle suggests that we cannot *handle* infinities, I admit that if we require a certain minimum time for each element of an infinite set, we cannot go through the infinite set, element by element, in finite time. However, this is not the only way to examine infinite sets. In fact, it is not even a possible way. Infinite sets must be treated in schematic ways, for example, with the help of (finite) descriptions of its (infinitely many) elements. Second, there is one particularly strong argument against actually infinite sets in Aristotle, which closely resembles the argument Aquinas constructs from the Book of Wisdom that was mentioned in Section 2: infinite quantities come in kinds according to the kinds of numbers. Because there is no kind of infinite numbers, there cannot be infinite quantities in either sets or magnitudes. However, Cantor's set theory has established exactly what Aristotle assumed to be impossible: kinds of infinite numbers. Hence, the abstract innumerability of infinite sets is no longer an argument against their possibility.

[29] We know this fact from modern set theory (take series up to $\omega + 1$, for example). The first to have realized this defect appears to be Bernard Bolzano, who considered movements through real space during a closed finite interval of time. Although such a movement has a first element, an infinite series of intermediate states is possible. See [Bolzano, 1834], part I, §68; and [Tapp, 2011a], 180.

6 Discrete Quantitative Infinity II: Resurrection Contra Aristotle[30]

The question regarding the eternity of the world was one of the most intricate issues for medieval philosophers and theologians. They discussed, for example, the following problem: if the world has existed for an infinite amount of time, then infinitely many revolutions of the celestial bodies have occurred. Hence, infinitely many revolutions have been actual.[31] However, how could that be, given that, according to Aristotle, infinite sets of actual things are impossible?

This problem can be solved fairly easily by adopting a weak form of presentism: only the present revolution is actual, whereas the infinitely many preceding revolutions are not. Scholastic philosophers distinguish sets that are "*in fieri*", or becoming, from sets that are "*in facto ens*", or existing at once.

The problem becomes much more difficult when Aristotelian philosophy is married to the Christian belief in resurrection. In traditional terms, the human soul continues to exist after death. However, if the thesis of the eternity of the world is sharpened to the thesis that there have been infinitely many generations before us, there is an actually infinite set that is not *in fieri* but *in facto ens*: the set of all souls. How could that be reconciled with Aristotle's ban on actual infinity?

Aquinas was reluctant to take refuge in the Christian belief that the world has existed for only a finite amount of time. He held that the world could well have existed from all eternity.[32] Although the problem did not force Christian philosophers to lift Aristotle's ban on actually infinite sets, it did exert some pressure to question the ban for the first time in its 1500-year history.

[30]This section and the following section of this paper have benefitted significantly from [Duhem, 1987], which is a selection from and translation of Duhem's masterpiece [Duhem, 1913–1959].

[31]This thesis was one that the archbishop of Paris solemnly condemned in 1277: "That there has already been an infinite number of revolutions of the heaven"; see [Fortin and O'Neill, 1963], [Uckelman, 2010] and [Flasch, 1989], 183.

[32]In a short treatise *De aeternitate mundi*, Aquinas explicitly says "*praetera non est adhuc demonstratum quod deus non possit facere ut sint infinita actu*" – "it has not yet been demonstrated that God cannot cause an infinite number of things to exist simultaneously" (translation from http://dhspriory.org/thomas/DeEternitateMundi.htm, 20.9.2013).

7 Continuous Quantitative Infinity: Omnipotence Contra Aristotle

Let us now proceed to the second part of Aristotle's ban: the impossibility of infinite magnitudes. Aristotle was even more restrictive with regard to magnitudes; not only was the existence *in actu* excluded, but even existing *in potentia* was excluded. The main reason for this exclusion was the following principle of philosophy of nature: if x exists potentially, it must be possible that x exists actually.[33] Let us call this principle "P". According to Aristotle, the spatial finitude of the world excluded an actually infinite magnitude, and this excluded something potentially infinitely large by principle P.

Principle P cannot be correct as it stands. For example, consider the Olympic Games. They exist potentially in two respects. First, at the moment when I write this article, the Olympics are not taking place. Second, if the Olympics took place, they would do so on a day-by-day basis but never be actual all at once. In the second respect, principle P is false: although the Olympic Games exist *in potentia*, they cannot become actual in the sense of being actual all at once.

Aquinas proposes an important amendment by suggesting that principle P is valid, but only in the realm of manipulating the actually existing parts of our universe.[34] Then, all that can potentially be combined can be combined only by using the actually existing building blocks. However, P does not hold for God. Because God is an omnipotent creator *ex nihilo*, his power is not restricted to manipulating already existing things.[35] A creator God can add an increasing number of stones to a pile by simply creating new stones from nothing. Therefore, he is not restricted by the finite amount of matter in the universe. Hence, principle P does not hold universally but only for

[33] A similar principle for possible non-existence plays an important role in Aquinas' *tertia via* (*STh* I,2,3). There, he claims that "*quod possibile est non esse, quandoque non est*" – "that which is possible not to be at some time is not" (*STh* I,2,3; transl. dhspriory.org, see above).

[34] Aquinas cites Aristotle that "*ex nihilo nihil fit*" was "a common axiom or opinion of the physicists ... because the natural agent, which was the object of their researches, does not act except by movement" (*De Pot.* 3,1; transl. dhspriory.org, see above).

[35] Aquinas indicates that when nature causes natural things, with regard to their form, nature presupposes matter; however, God causes the things "*ex nihilo*" (*STh* I,45,2 c.a.).

intramundane agents.[36]

Reasoning of this type leads to the next Aristotelian ban being lifted: the assumption of an omnipotent creator God finally destroyed Aristotle's arguments regarding the impossibility of a potentially infinite magnitude.

8 A Link Between Essential and Quantitative Infinity

Thus far, we have analyzed the various meanings of "infinity" that are important in Aquinas' doctrine of God. These meanings vary greatly: essential infinity means that there is no *contractio amplitudinis* of a form by matter; discrete quantitative infinity means a type of quantitative infinity quite familiar to us from the natural numbers; and infinite power means power that has no limitations. Are there conceptual connections between these meanings, or are they completely different concepts that are only equivocally named by the same word, "infinity"?

A simple connection exists between discrete quantitative infinity and extensional infinity of power in that a power is called "infinite" if it extends to infinitely many effects. Aquinas himself traces this conceptual connection in the context of his theory of Christ's soul. Aquinas is concerned with the following tension: as a creature, a human being, Christ must have a finite soul, but as the incarnation of the second person of the Trinity, he must share God's infinity. Aquinas solves this tension regarding divine power by distinguishing between extensional and intensional infinity of power. A creature can have infinite power in so far as the power extends, for example, to all possibilities of beings, which are infinitely many.[37]

This first conceptual connection is only an application; the concept of discrete quantitative infinity is used to state what exactly infinite power consists in. May one consider this as evidence regarding the basicality of the concept of quantitative infinity?

[36] In *STh* I,45,2 ad 1, Aquinas considers the ancient principle "*ex nihilo nihil fieri*" and relativizes its validity by suggesting that the ancient philosophers "considered only the emanation of particular effects from particular causes" when this principle came to their minds. In *De Pot* 3,1, Aquinas explicitly states that a subject that is necessary for (mundane) movement or change "is not required for a supernatural agent" (transl. dhspriory.org, see above).

[37] *Super Sent.*, lib. 3 d. 14 q. 1 a. 2 qc. 2.

Another interesting conceptual connection is also drawn by Aquinas himself; however, this time, it is drawn from revelational theology proper and not from philosophical theology. In his *Commentary on the Gospel of John*, Aquinas must explain the verse that Christ is "full of grace and truth" (John 1:14). How much grace does Christ have? Is it enough for all people on earth who starve for divine grace? Certainly, suggests Aquinas, because Christ merits "through his teaching and works and the sufferings of his death, superabundant grace for an infinite number of worlds, if there were such."[38] Why can man not arrive at this status of grace all by himself? Because such a status would mean a perfect union with God, and such a union would require perfectly loving the infinite good. However, finite beings cannot perfectly love the infinite good.

This is a particularly interesting line of thought because it is one of the few in which Aquinas explicitly links both a quantitative and a qualitative or metaphysical concept of infinity: Christ's grace would cover infinitely many worlds (if there were such), which is the quantitative side, and we need that grace because we are, in a metaphysical sense, finite creatures who cannot adequately relate to the infinite good.

9 Summary and Outlook

According to Aquinas, only God is essentially infinite. All other beings are finite *simpliciter* and can be infinite only *secundum quid*. God alone is pure form, unlimited by matter, and a pure unreceived being not determined by *genus* and *species*. His infinity is not quantitative; the infinity predicate is not used syncategorematically with regard to multitude or magnitude but categorematically. Hence, Aristotle's negative outlook on infinity does not bear upon God's existence directly. Instead, its validity is restricted to the realm of nature/creation.

Nevertheless, there are tensions between Aristotle's bans on actually infinite multitudes and potentially infinite magnitudes on the one hand and Christian belief on the other hand. The first problem concerns actually infinite multitudes and Christian belief in resurrection. In the end, watering down Aristotle's ban of the actually infinite multitude seemed to be the most natural way to address the problem that – in the (possible) case of a backwardly

[38] *Super Io* L.1 l.8.

eternal world with an infinite number of human beings having lived, having died and having been preserved for resurrection – there would be an infinite multitude of actually existing souls. The second problem concerns Aristotle's shift from banning actually infinite magnitudes to banning potentially infinite magnitudes. This move was justified by a principle P, which Aquinas argues is plausible only for intermundane agents, whose actions require some matter to act upon. However, this principle is not valid for a world-transcendent omnipotent God, who can create "*ex nihilo*". Hence, the belief in the Christian God casts doubts on Aristotle's ban against potentially infinite magnitudes even if one maintains the finitude of the physical universe and therefore the impossibility of infinitely large actual things.

Infinity also plays other important roles in the doctrine of God. Quantitative infinity in the sense of a *multitude* is important both in connection with infinite regress arguments and in the confrontation of the Christian belief in resurrection with Aristotle's ban on actually infinite magnitudes and his belief in the eternity of the world. In Aquinas' eyes, quantitative infinity in the sense of a *magnitude*, which Aristotle believed to be impossible even *in potentia*, is implied by divine omnipotence, which includes not only the manipulation of the finite existing amount of matter in the universe but also the ability to create new matter *ex nihilo*.

Finally, we have noted some connections between different senses of "infinity". For example, essential divine infinity precludes being completely known or fully loved by finite creatures, and the essential infinity of Christ's grace would be sufficient even for the salvation of quantitatively infinite worlds.

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